# Introduction to Information Retrieval

Introducing ranked retrieval

#### Ranked retrieval

- Thus far, our queries have all been Boolean.
  - Documents either match or don't.
- Good for expert users with precise understanding of their needs and the collection.
  - Also good for applications: Applications can easily consume 1000s of results.
- Not good for the majority of users.
  - Most users incapable of writing Boolean queries (or they are, but they think it's too much work).
  - Most users don't want to wade through 1000s of results.
    - This is particularly true of web search.

### Problem with Boolean search: feast or famine

- Boolean queries often result in either too few (≈0) or too many (1000s) results.
  - Query 1: "standard user dlink 650"  $\rightarrow$  200,000 hits
  - Query 2: "standard user dlink 650 no card found" → 0 hits
- It takes a lot of skill to come up with a query that produces a manageable number of hits.
  - AND gives too few; OR gives too many

#### Ranked retrieval models

- Rather than a set of documents satisfying a query expression, in ranked retrieval models, the system returns an ordering over the (top) documents in the collection with respect to a query
- Free text queries: Rather than a query language of operators and expressions, the user's query is just one or more words in a human language
- In principle, there are two separate choices here, but in practice, ranked retrieval models have normally been associated with free text queries and vice versa

### Feast or famine: not a problem in ranked retrieval

- When a system produces a ranked result set, large result sets are not an issue
  - Indeed, the size of the result set is not an issue
  - We just show the top k (  $\approx$  10) results
  - We don't overwhelm the user
  - Premise: the ranking algorithm works

#### Scoring as the basis of ranked retrieval

- We wish to return in order the documents most likely to be useful to the searcher
- How can we rank-order the documents in the collection with respect to a query?
- Assign a score say in [0, 1] to each document
- This score measures how well document and query "match".

#### Query-document matching scores

- We need a way of assigning a score to a query/ document pair
- Let's start with a one-term query
- If the query term does not occur in the document: score should be 0
- The more frequent the query term in the document, the higher the score (should be)
- We will look at a number of alternatives for this

# Introduction to Information Retrieval

Introducing ranked retrieval

# Introduction to Information Retrieval

Scoring with the Jaccard coefficient

#### Take 1: Jaccard coefficient

- A commonly used measure of overlap of two sets A and B is the Jaccard coefficient
- jaccard(A,B) =  $|A \cap B| / |A \cup B|$
- jaccard(A,A) = 1
- jaccard(A,B) = 0 if  $A \cap B = 0$
- A and B don't have to be the same size.
- Always assigns a number between 0 and 1.

#### Jaccard coefficient: Scoring example

- What is the query-document match score that the Jaccard coefficient computes for each of the two documents below?
- Query: ides of march
- Document 1: caesar died in march
- Document 2: the long march

#### Issues with Jaccard for scoring

- It doesn't consider term frequency (how many times a term occurs in a document)
  - Rare terms in a collection are more informative than frequent terms
  - Jaccard doesn't consider this information
- We need a more sophisticated way of normalizing for length
  - Later in this lecture, we'll use  $|A \cap B|/\sqrt{|A \cup B|}$  . . . instead of  $|A \cap B|/|A \cup B|$  (Jaccard) for length normalization.

# Introduction to Information Retrieval

Scoring with the Jaccard coefficient

# Introduction to Information Retrieval

Term frequency weighting

### Recall: Binary term-document incidence matrix

	<b>Antony and Cleopatra</b>	<b>Julius Caesar</b>	The Tempest	Hamlet	Othello	Macbeth
Antony	1	1	0	0	0	1
Brutus	1	1	0	1	0	0
Caesar	1	1	0	1	1	1
Calpurnia	0	1	0	0	0	0
Cleopatra	1	0	0	0	0	0
mercy	1	0	1	1	1	1
worser	1	0	1	1	1	0

Each document is represented by a binary vector  $\in \{0,1\}^{|V|}$ 

#### Term-document count matrices

- Consider the number of occurrences of a term in a document:
  - Each document is a count vector in  $\mathbb{N}^{|V|}$ : a column below

	<b>Antony and Cleopatra</b>	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	157	73	0	0	0	0
Brutus	4	157	0	1	0	0
Caesar	232	227	0	2	1	1
Calpurnia	0	10	0	0	0	0
Cleopatra	57	0	0	0	0	0
mercy	2	0	3	5	5	1
worser	2	0	1	1	1	0

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#### Bag of words model

- Vector representation doesn't consider the ordering of words in a document
- John is quicker than Mary and Mary is quicker than
   John have the same vectors

- This is called the bag of words model.
- In a sense, this is a step back: The positional index was able to distinguish these two documents
  - We will look at "recovering" positional information later on
  - For now: bag of words model

#### Term frequency tf

- The term frequency  $tf_{t,d}$  of term t in document d is defined as the number of times that t occurs in d.
- We want to use tf when computing query-document match scores. But how?
- Raw term frequency is not what we want:
  - A document with 10 occurrences of the term is more relevant than a document with 1 occurrence of the term.
  - But not 10 times more relevant.
- Relevance does not increase proportionally with term frequency.

NB: frequency = count in IR

### Log-frequency weighting

The log frequency weight of term t in d is

$$w_{t,d} = \begin{cases} 1 + \log_{10} tf_{t,d}, & \text{if } tf_{t,d} > 0 \\ 0, & \text{otherwise} \end{cases}$$

Score for a document-query pair: sum over terms t in both q and d:

• score = 
$$\sum_{t \in q \cap d} (1 + \log t f_{t,d})$$

The score is 0 if none of the query terms is present in the document.

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- $0 \to 0, 1 \to 1, 2 \to 1.3, 10 \to 2, 1000 \to 4$ , etc.
- Score for a document-query pair: sum over terms t in both q and d:

• score = 
$$\sum_{t \in q \cap d} (1 + \log t f_{t,d})$$

The score is 0 if none of the query terms is present in the document.

# Introduction to Information Retrieval

Term frequency weighting

# Introduction to Information Retrieval

(Inverse) Document frequency weighting

#### Document frequency

- Rare terms are more informative than frequent terms
  - Recall stop words
- Consider a term in the query that is rare in the collection (e.g., arachnocentric)
- A document containing this term is very likely to be relevant to the query arachnocentric
- → We want a high weight for rare terms like arachnocentric.

#### Document frequency, continued

- Frequent terms are less informative than rare terms
- Consider a query term that is frequent in the collection (e.g., high, increase, line)
- A document containing such a term is more likely to be relevant than a document that doesn't
- But it's not a sure indicator of relevance.
- → For frequent terms, we want positive weights for words like high, increase, and line
- But lower weights than for rare terms.
- We will use document frequency (df) to capture this.

### idf weight

- df<sub>t</sub> is the <u>document</u> frequency of t: the number of documents that contain t
  - df<sub>t</sub> is an inverse measure of the informativeness of t
  - $df_t \leq N$
- We define the idf (inverse document frequency) of t
   by

$$idf_t = \log_{10} (N/df_t)$$

• We use  $\log (N/df_t)$  instead of  $N/df_t$  to "dampen" the effect of idf.

Will turn out the base of the log is immaterial.

#### idf example, suppose N = 1 million

term	$df_t$	idf <sub>t</sub>
calpurnia	1	
animal	100	
sunday	1,000	
fly	10,000	
under	100,000	
the	1,000,000	

$$idf_t = log_{10} (N/df_t)$$

There is one idf value for each term t in a collection.

### Effect of idf on ranking

- Question: Does idf have an effect on ranking for oneterm queries, like
  - iPhone

#### Effect of idf on ranking

- Question: Does idf have an effect on ranking for oneterm queries, like
  - iPhone
- idf has no effect on ranking one term queries
  - idf affects the ranking of documents for queries with at least two terms
  - For the query capricious person, idf weighting makes occurrences of capricious count for much more in the final document ranking than occurrences of person.

#### Collection vs. Document frequency

 The collection frequency of t is the number of occurrences of t in the collection, counting multiple occurrences.

#### Example:

Word	Collection frequency	Document frequency
insurance	10440	3997
try	10422	8760

Which word is a better search term (and should get a higher weight)?

# Introduction to Information Retrieval

(Inverse) Document frequency weighting

# Introduction to Information Retrieval

tf-idf weighting

### tf-idf weighting

 The tf-idf weight of a term is the product of its tf weight and its idf weight.

$$\mathbf{w}_{t,d} = (1 + \log t \mathbf{f}_{t,d}) \times \log_{10}(N/d\mathbf{f}_t)$$

- Best known weighting scheme in information retrieval
  - Note: the "-" in tf-idf is a hyphen, not a minus sign!
  - Alternative names: tf.idf, tf x idf
- Increases with the number of occurrences within a document
- Increases with the rarity of the term in the collection

### Final ranking of documents for a query

$$Score(q,d) = \sum_{t \in q \cap d} tf.idf_{t,d}$$

#### Binary $\rightarrow$ count $\rightarrow$ weight matrix

	Antony and Cleopatra	Julius Caesar	The Tempest	Hamlet	Othello	Macbeth
Antony	5.25	3.18	0	0	0	0.35
Brutus	1.21	6.1	0	1	0	0
Caesar	8.59	2.54	0	1.51	0.25	0
Calpurnia	0	1.54	0	0	0	0
Cleopatra	2.85	0	0	0	0	0
mercy	1.51	0	1.9	0.12	5.25	0.88
worser	1.37	0	0.11	4.15	0.25	1.95

Each document is now represented by a real-valued vector of tf-idf weights  $\in \mathbb{R}^{|V|}$ 

# Introduction to Information Retrieval

tf-idf weighting

# Introduction to Information Retrieval

The Vector Space Model (VSM)

#### Documents as vectors

- Now we have a |V|-dimensional vector space
- Terms are axes of the space
- Documents are points or vectors in this space
- Very high-dimensional: tens of millions of dimensions when you apply this to a web search engine
- These are very sparse vectors most entries are zero

#### Queries as vectors

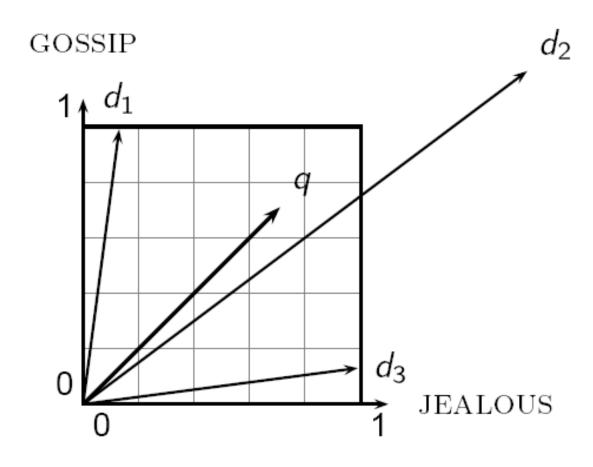
- Key idea 1: Do the same for queries: represent them as vectors in the space
- Key idea 2: Rank documents according to their proximity to the query in this space
- proximity = similarity of vectors
- proximity ≈ inverse of distance
- Recall: We do this because we want to get away from the you're-either-in-or-out Boolean model
- Instead: rank more relevant documents higher than less relevant documents

#### Formalizing vector space proximity

- First cut: distance between two points
  - ( = distance between the end points of the two vectors)
- Euclidean distance?
- Euclidean distance is a bad idea . . .
- ... because Euclidean distance is large for vectors of different lengths.

#### Why distance is a bad idea

The Euclidean distance between q and  $\overrightarrow{d_2}$  is large even though the distribution of terms in the query  $\overrightarrow{q}$  and the distribution of terms in the document  $\overrightarrow{d}_2$  are very similar.



#### Use angle instead of distance

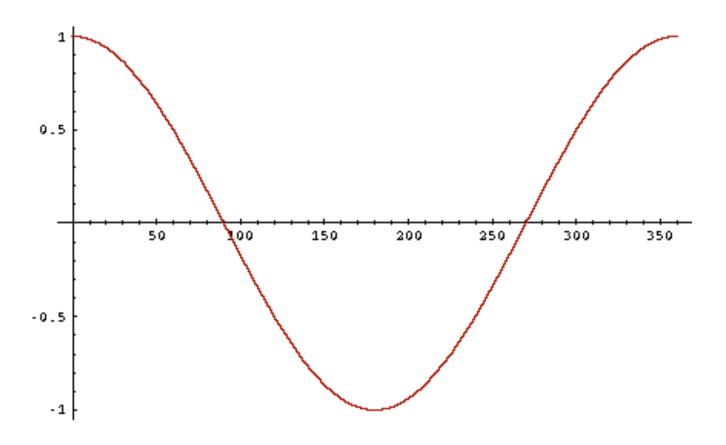
- Thought experiment: take a document d and append it to itself. Call this document d'.
- "Semantically" d and d' have the same content
- The Euclidean distance between the two documents can be quite large
- The angle between the two documents is 0, corresponding to maximal similarity.

 Key idea: Rank documents according to angle with query.

#### From angles to cosines

- The following two notions are equivalent.
  - Rank documents in <u>decreasing</u> order of the angle between query and document
  - Rank documents in <u>increasing</u> order of cosine(query,document)
- Cosine is a monotonically decreasing function for the interval [0°, 180°]

### From angles to cosines



But how – and why – should we be computing cosines?

#### Length normalization

- A vector can be (length-) normalized by dividing each of its components by its length for this we use the  $L_2$  norm:  $\|\vec{x}\|_2 = \sqrt{\sum_i x_i^2}$
- Dividing a vector by its L<sub>2</sub> norm makes it a unit (length) vector (on surface of unit hypersphere)
- Effect on the two documents d and d' (d appended to itself) from earlier slide: they have identical vectors after length-normalization.
  - Long and short documents now have comparable weights

### cosine(query,document)

Dot product
$$\cos(\vec{q}, \vec{d}) = \frac{\vec{q} \cdot \vec{d}}{|\vec{q}||\vec{d}|} = \frac{\vec{q}}{|\vec{q}|} \cdot \frac{\vec{d}}{|\vec{d}|} = \frac{\sum_{i=1}^{|V|} q_i d_i}{\sqrt{\sum_{i=1}^{|V|} q_i^2} \sqrt{\sum_{i=1}^{|V|} d_i^2}}$$

 $q_i$  is the tf-idf weight of term i in the query  $d_i$  is the tf-idf weight of term i in the document

 $\cos(\vec{q}, \vec{d})$  is the cosine similarity of  $\vec{q}$  and  $\vec{d}$  ... or, equivalently, the cosine of the angle between  $\vec{q}$  and  $\vec{d}$ .

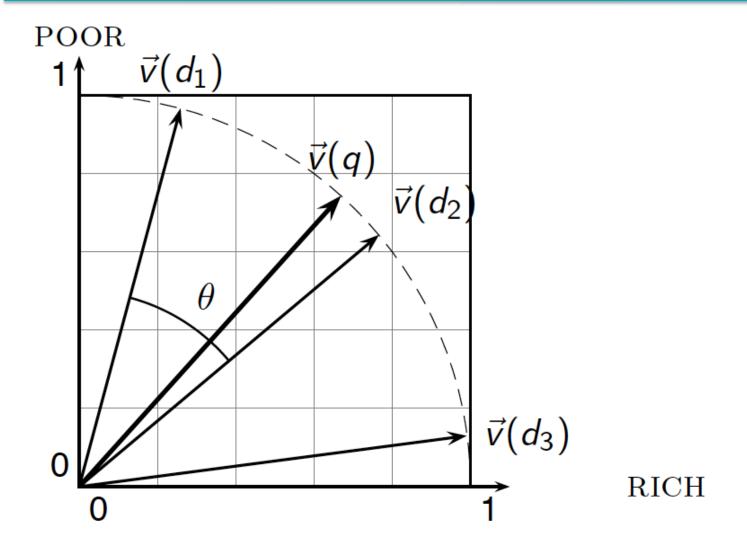
#### Cosine for length-normalized vectors

For length-normalized vectors, cosine similarity is simply the dot product (or scalar product):

$$\cos(\vec{q}, \vec{d}) = \vec{q} \cdot \vec{d} = \sum_{i=1}^{|V|} q_i d_i$$

for q, d length-normalized.

### Cosine similarity illustrated



#### Cosine similarity amongst 3 documents

How similar are

the novels

SaS: Sense and

Sensibility

PaP: Pride and

Prejudice, and

**WH**: Wuthering

Heights?

term	SaS	PaP	WH
affection	115	58	20
jealous	10	7	11
gossip	2	0	6
wuthering	0	0	38

Term frequencies (counts)

Note: To simplify this example, we don't do idf weighting.

#### 3 documents example contd.

#### Log frequency weighting

#### **After length normalization**

term	SaS	PaP	WH
affection	3.06	2.76	2.30
jealous	2.00	1.85	2.04
gossip	1.30	0	1.78
wuthering	0	0	2.58

term	SaS	PaP	WH
affection	0.789	0.832	0.524
jealous	0.515	0.555	0.465
gossip	0.335	0	0.405
wuthering	0	0	0.588

```
cos(SaS,PaP) \approx
0.789 \times 0.832 + 0.515 \times 0.555 + 0.335 \times 0.0 + 0.0 \times 0.0 \approx 0.94
cos(SaS,WH) \approx 0.79
cos(PaP,WH) \approx 0.69
```

Why do we have cos(SaS,PaP) > cos(SAS,WH)?

# Introduction to Information Retrieval

The Vector Space Model (VSM)

## Introduction to **Information Retrieval**

Calculating tf-idf cosine scores in an IR system

### tf-idf weighting has many variants

Term frequency		Docum	ent frequency	Normalization		
n (natural)	$tf_{t,d}$	n (no)	1	n (none)	1	
I (logarithm)	$1 + \log(tf_{t,d})$	t (idf)	$\log \frac{N}{\mathrm{df_t}}$	c (cosine)	$\frac{1}{\sqrt{w_1^2 + w_2^2 + \dots + w_M^2}}$	
a (augmented)	$0.5 + \frac{0.5 \times tf_{t,d}}{max_t(tf_{t,d})}$	p (prob idf)	$\max\{0,\log \frac{N-\mathrm{df}_t}{\mathrm{df}_t}\}$	u (pivoted unique)	1/u	
b (boolean)	$\begin{cases} 1 & \text{if } \operatorname{tf}_{t,d} > 0 \\ 0 & \text{otherwise} \end{cases}$			b (byte size)	$1/\mathit{CharLength}^{lpha}$ , $lpha < 1$	
L (log ave)	$\frac{1 + \log(\operatorname{tf}_{t,d})}{1 + \log(\operatorname{ave}_{t \in d}(\operatorname{tf}_{t,d}))}$					

Columns headed 'n' are acronyms for weight schemes.

Why is the base of the log in idf immaterial?

### tf-idf weighting has many variants

Term frequency		Docum	ent frequency	Normalization		
n (natural)	$tf_{t,d}$	n (no)	1	n (none)	1	
	$1 + \log(tf_{t,d})$	t (idf)	$\log \frac{N}{\mathrm{df_t}}$	c (cosine)	$\frac{1}{\sqrt{w_1^2 + w_2^2 + \dots + w_M^2}}$	
a (augmented)	$0.5 + \frac{0.5 \times tf_{t,d}}{max_t(tf_{t,d})}$	p (prob idf)	$\max\{0,\log \frac{N-\mathrm{df}_t}{\mathrm{df}_t}\}$	u (pivoted unique)	1/u	
b (boolean)	$\begin{cases} 1 & \text{if } \operatorname{tf}_{t,d} > 0 \\ 0 & \text{otherwise} \end{cases}$			b (byte size)	$1/\mathit{CharLength}^{lpha}, \ lpha < 1$	
L (log ave)	$\frac{1 + \log(\operatorname{tf}_{t,d})}{1 + \log(\operatorname{ave}_{t \in d}(\operatorname{tf}_{t,d}))}$					

### Weighting may differ in queries vs documents

- Many search engines allow for different weightings for queries vs. documents
- SMART Notation: denotes the combination in use in an engine, with the notation ddd.qqq, using the acronyms from the previous table
- A very standard weighting scheme is: Inc.ltc
- Document: logarithmic tf (l as first character), no idf and cosine normalization
  A bad idea?
- Query: logarithmic tf (l in leftmost column), idf (t in second column), cosine normalization ...

#### tf-idf example: Inc.ltc

Document: car insurance auto insurance

Query: best car insurance

Term	Query					Document				Prod	
	tf- raw	tf-wt	df	idf	wt	n' liz e	tf-raw	tf-wt	wt	n' liz e	
auto	0	0	5000	2.3	0	0	1	1	1	0.52	0
best	1	1	50000	1.3	1.3	0.34	0	0	0	0	0
car	1	1	10000	2.0	2.0	0.52	1	1	1	0.52	0.27
insurance	1	1	1000	3.0	3.0	0.78	2	1.3	1.3	0.68	0.53

Exercise: what is *N*, the number of docs?

Doc length = 
$$\sqrt{1^2 + 0^2 + 1^2 + 1.3^2} \approx 1.92$$

Score = 
$$0+0+0.27+0.53 = 0.8$$

#### Computing cosine scores

```
CosineScore(q)
     float Scores[N] = 0
  2 float Length[N]
  3 for each query term t
    do calculate w_{t,q} and fetch postings list for t
         for each pair(d, tf<sub>t,d</sub>) in postings list
         do Scores[d] += w_{t,d} \times w_{t,q}
  6
     Read the array Length
     for each d
  8
     do Scores[d] = Scores[d]/Length[d]
     return Top K components of Scores[]
 10
```

### Summary – vector space ranking

- Represent the query as a weighted tf-idf vector
- Represent each document as a weighted tf-idf vector
- Compute the cosine similarity score for the query vector and each document vector
- Rank documents with respect to the query by score
- Return the top K (e.g., K = 10) to the user

## Introduction to **Information Retrieval**

Calculating tf-idf cosine scores in an IR system

# Introduction to Information Retrieval

Evaluating search engines

#### Measures for a search engine

- How fast does it index
  - Number of documents/hour
  - (Average document size)
- How fast does it search
  - Latency as a function of index size
- Expressiveness of query language
  - Ability to express complex information needs
  - Speed on complex queries
- Uncluttered UI
- Is it free?

#### Measures for a search engine

- All of the preceding criteria are measurable: we can quantify speed/size
  - we can make expressiveness precise
- The key measure: user happiness
  - What is this?
  - Speed of response/size of index are factors
  - But blindingly fast, useless answers won't make a user happy
- Need a way of quantifying user happiness with the results returned
  - Relevance of results to user's information need

#### Evaluating an IR system

- An information need is translated into a query
- Relevance is assessed relative to the information need not the query
- E.g., <u>Information need</u>: I'm looking for information on whether drinking red wine is more effective at reducing your risk of heart attacks than white wine.
- Query: wine red white heart attack effective
- You evaluate whether the doc addresses the information need, not whether it has these words

#### **Evaluating ranked results**

- Evaluation of a result set:
  - If we have
    - a benchmark document collection
    - a benchmark set of queries
    - assessor judgments of whether documents are relevant to queries

Then we can use Precision/Recall/F measure as before

- Evaluation of ranked results:
  - The system can return any number of results
  - By taking various numbers of the top returned documents (levels of recall), the evaluator can produce a precisionrecall curve

#### Recall/Precision

R P

- 1 R
- **2** N
- **3** N
- 4 R
- 5 R
- 6 N
- 7 R
- 8 N
- **9** N
- **10** N

Assume 10 rel docs in collection

#### Two current evaluation measures...

- Mean average precision (MAP)
  - AP: Average of the precision value obtained for the top k documents, each time a relevant doc is retrieved
  - Avoids interpolation, use of fixed recall levels
  - Does weight most accuracy of top returned results
  - MAP for set of queries is arithmetic average of APs
    - Macro-averaging: each query counts equally

# Introduction to Information Retrieval

Evaluating search engines